Technical Notes

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A Model for Unsteady Transonic Indicial Responses

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THE importance of the accurate prediction of unsteady aerodynamic loads in transonic flight is reflected in the many recently developed codes to compute these loads. Among them, in two dimensions, are the Euler code of Magnus and Yoshihara,[1] the full potential code of Chipman and Jameson,[2,3] and of Goorjian,[4] the small perturbation potential code of Ballhaus and Goorjian,[5] the time-linearized small perturbation code of Fung et al.,[6] and the harmonic small perturbation potential code of Ehlers,[7] and of Traci et al.[8] In three dimensions we now have the very recent small perturbation code of Borland et al.[9] and the full potential code of Steger and Caradonna.[10]

A crucial problem is the calculation of flutter boundaries in the transonic regime.[11] If the above codes are coupled with linear structural codes to predict flutter boundaries, the computational cost can be quite high. Even the two-dimensional study by Yang et al.[12] of the flutter boundaries for the NACA 64A006 airfoil required 2.4 h of CDC 6500 time. Aerodynamic results found are not the same as Yang et al., but the computational time to find flutter boundaries is indicative, and we note again here that shock motions must be included in such studies.[13] A simple method of characterizing the aerodynamic forces that can greatly reduce the cost of aeroelastic studies is proposed.

With the assumption of time-linear behavior, an indicial response can represent the response of all frequencies. The computational experiment of Ballhaus and Goorjian[14] and wind-tunnel experiment of Davis and Malcolm[15] support the validity of this approach provided shock excursions remain a small fraction of the chord. Thus, the method should be applicable in most flutter studies.

A typical indicial response curve possesses a very distinctive time scale. Nixon[16] tried to develop a theory of a universal indicial function but only succeeded in describing it adequately for a very short time interval. Here a means of characterizing the indicial curve by a time scale parameter λ is illustrated and the values of such a characterization are shown.

It is assumed that for small amplitude oscillations the time linearization of the unsteady potential equation is adequate. The perturbed potential $\phi(x,y,t)$ about a steady flow characterized by $\phi^\circ(x,y)$ satisfies the equation

$$-2k\phi_{xt}+\{(K-\phi_x^\circ)\phi\}_x+\phi_{yy}=0$$

and the appropriate boundary conditions (see Refs. 6 and 17). This equation is solved numerically for the indicial motion of a given mode. The resultant aerodynamic functions $C_\ell(t)$ and $C_m(t)$, i.e., the lift and moment coefficients produced by a step change in incidence or control surface deflection, can be superimposed to produce motion of any kind and, in particular, harmonic motion of reduced frequency k.

By sampling a number of indicial curves, one concludes that the aerodynamic coefficients $C(t)$ are well described by the equation

$$\dot{C}+\lambda C=0 \tag{1}$$

where λ characterizes the time scale of the response. An easy way to choose λ is to minimize the error functional

$$I(\lambda)=\int_0^T(\dot{C}_I+\lambda C_I)^2\mathrm{d}t$$

with respect to λ; $C_I(t)$ is the computed indicial response normalized by its asymptotic value $C_{\max}$; and T is a sufficiently large time for $C_I(t)$ to have reached its asymptotic value. Choosing λ to minimize I gives

$$\lambda=-\frac{C_I^2(T)-C_I^2(0)}{2\int_0^T C_I^2\mathrm{d}t} \tag{2}$$

Thus, all indicial responses are approximated by $C_{\max}(1-e^{-\lambda t})$. $C(t)$ may be thought of as being the first approximation of $C_I(t)$ which is modeled by a sequence of harmonic oscillators.

The amplitude and phase shift due to harmonic motion can then be calculated by the Duhamel integral, i.e.,

$$C(t,k)=C_{\max}\int_0^T\{1-e^{-\lambda(t-\hat{t})}\}\frac{\mathrm{d}}{\mathrm{d}\hat{t}}\sin k\hat{t}\,\mathrm{d}\hat{t}$$

$$=A\sin(kt-\theta) \tag{3}$$

where

$$A=\frac{C_{\max}}{\left[1+\left(\frac{k}{\lambda}\right)^2\right]^{1/2}}\quad\text{and}\quad\theta=\tan^{-1}\left(\frac{k}{\lambda}\right)$$

It is discovered that if the response can be modeled in this way, then the single parameter (k/λ) determines the dependence of the amplitude and phase shift on k. Figures 1 and 2 compare the values of amplitude and phase determined by direct computation with those obtained from this modeling. The agreement is generally good for reduced frequencies less than 0.2. The discrepancy due to this simple modeling at higher reduced frequencies is inevitable; nevertheless, the strong dependence on k/λ remains evident in Figs. 1 and 2. Figure 3 depicts the variation of $C_{\max}$ and λ with Mach number for the NACA 64A006 airfoil.

The result here suggests that at sufficiently low reduced frequencies it is possible to represent an indicial response curve with just two parameters $C_{\max}$ and λ. Such a simplification, used with any linear structural model, can greatly reduce the computational cost of finding flutter boundaries. The extension of these ideas to the next order, viz.,

$$C_I(t)=C(t)+D(t)+\ldots$$

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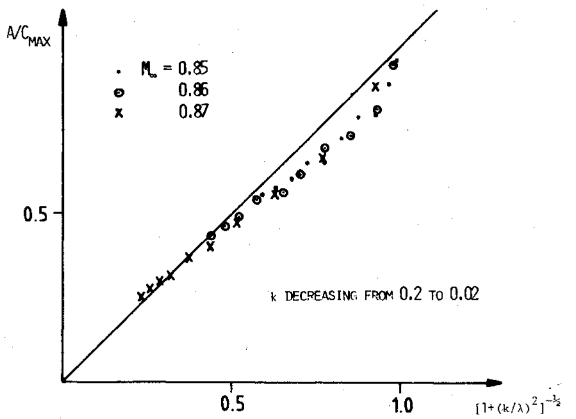

Fig. 1 Comparison of the variation of $A/C_{\max}$ in Eq. (3) (solid line) with the variation of computed values of the lift coefficient for the NACA 64A006 airfoil pitching about midchord at zero mean angle of attack.

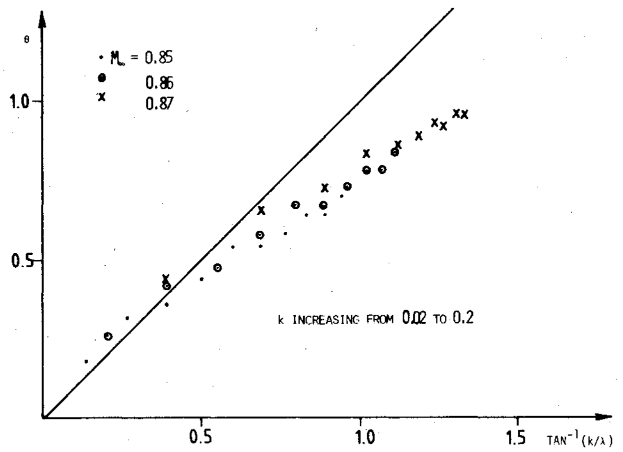

Fig. 2 Comparison of the variation of θ in Eq. (3) (solid line) with the variation of computed values of the lift coefficient for the NACA 64A006 airfoil pitching about midchord at zero mean angle of attack.

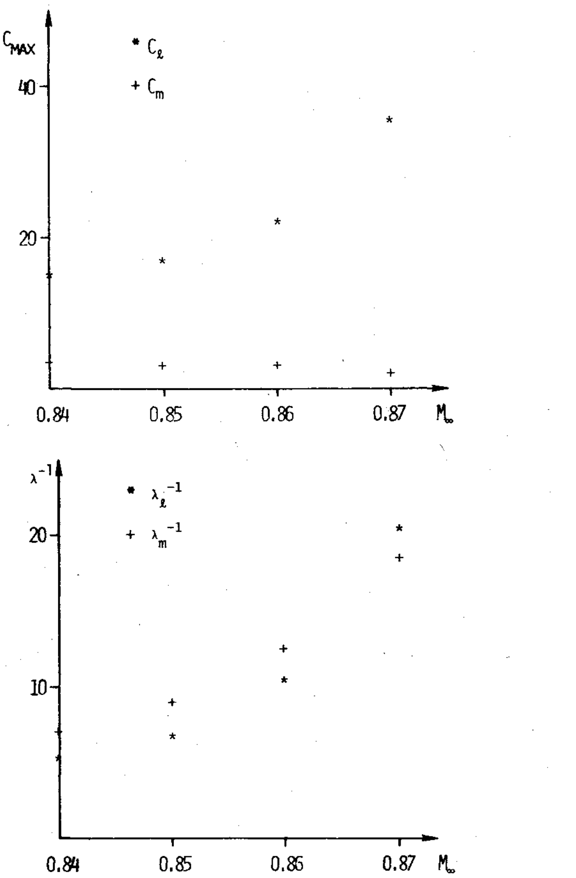

Fig. 3 Variation of $C_{\max}$ and λ with Mach number for the NACA 64A006 airfoil pitching about midchord at zero mean angle of attack.

where $\ddot{D}+\alpha\dot{D}+\beta D=0$ should extend the applicability of the method.[13]

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